\section{Testing means}

Linear combination

We consider the so called null hypothesis:

$H\_o: \mu = \mu \_o$

Where $\mu$ is unknown but true value of the mean while $\mu\_o$ is a known hypothesized value of the true mean.

The significance level is a small value close to zero often expressed in percentage for example 5%. It represent the percentage of mistakenly rejecting $H\_o$ hypothesis that we can afford.

Test and example

.$.¤..,.u,m¤L...V.....m.. ,,..,.,.,1W,.`m{..`.4... X;C (5 0 \_ 4 , 5 O . 7 , 4 9 . 1 , 4 9 . O , 5 1 . 1)

wxm X — >.m.p4K mw., 1 Wm `umwm .m·`.mw me Hemvnxwx WQEH ( X )

s d ( x )

EBU; §.€5¥'§I?J.`C§2Q§Z? 2’7;,‘1¥ZS§T;1‘i‘2iL°!2l'J"§ C - t G S C ( X V mu; 5 0 )

# One Sample t—test

F\*¤·"¤‘·· A 2 \* —

an ¤ nm mum M ¤m.........g .<k,.m.. ww, me ¤»vv.»mg uvm # dat .3 ; X

W W, M, M, m»,`..`· #t = O . 1404, df = 4 , p-value = O . 89

my #e1te1rnat ive hypethes i sz t rue mean

H7Z$T.iZ.£'.$?v;3fE2`JT`i?J@,i'ZY`.2’I,XZ§E?§L2Zl1`?l;2§ # 9 5 pe Ice ¤ C c on f i den ce i nt e Iva 1 =

mu. ,4\*... A > me `r.` M. F``. A M <,»:.».»., me wm. # s amp l e e s t i mat e s 2

.> M m¤¤...¤ U. .,m»,¤.— wc. Nm .,1w mu my me nm mm mn #m€,3y; O f X

# 5 O . O 6

\begin{verbatim}

x=c(50.4,50.7,49.1,49.0,51.1)

mean(x)

sd(x)

t.test(x,mu=50)

# One Sample t-test

#data: x

#t = 0.1404, df = 4, p-value = 0.8951

#alternative hypothesis: true mean #95 percent confidence interval:

# 48.87358 51.24642

#sample estimates:

#mean of x

# 50.06

\end{verbatim}

Comparison of two experimental means

We consider a null hypothesis of the form:

We consider a null hypothesis of the form:

\[H0 : \mu\_1 =\mu \_2\]

where $\mu\_1$ and $\mu\_2$ are hypothesized values of the means for two

populations.

The goal is to determine if there is enough support in the data

to claim that the means are not equal..

Another way in which the results of a new analytical methods

may be tested in by comparing them with whose obtained by a second (perhaps a reference method)

In this case we have two sample means \bar{X}\_1 and \bar{X}\_2.

Taking the null hypothesis that the two methods give the same result, that is

\[H\_o: \mu\_1 = \mu\_2\] we need to test

7 E,—ig differs significantly from zero. lf the two sample: have standard \_' which are not significantly different (see Section 5.5 for a method 0

\_ dis assumption), a pooled estimate, s, ul the standard deviation can be

.' .¤ Emu! the two individual standard deviations t, and 2,.

In order to decide whether the difference between two sample rueam it and

significanct, that is to test the null hypothesis, H": A = yi, Lhe stalistlt t xs

\[ t\_{TS} = \frac{\bar{x}\_1 – \bar{x}\_2}{s \sqrt{\frac{1}{n\_1} + \frac{1}{n\_2}}} \]where $s$ is calculated from:

\[ s^2 = \frac{(n\_1-1)s^2\_1 +( n\_2-1)s^2\_2 }{n\_1+n\_2 -2} \]

and t has vt, + ni - 2 degrees of freedom.

Tlns method assumes that the samples are drawn from populations with equal

standard deviation

In a comparison of two methods for the determination of chromium in rye

mass, the following results (mg kg^{-1}Cr) were obtained:

Method I: mean : 1.48; standard deviation 0.28

Method 2: mean : 2.33; standard deviation 0.31

For each method five determinations were made.

(Sahuquillo, A., Rubio, R. and Raurct, G, 1999, Anulysl 12A; 1)

Do these two methods give results having means which differ significantly!

The null hypothesis adopted is that the means of the results given by the two

methods are equal, From equation (3.3), the pooled value of the standard deviation is given by-

\[s2= \frac{ ([4 \times 0.28^2]) + [4 \times 0.31^2])}{(5 + 5 -2} =0.0873 \]

$s = 0.295$

From equation (3.2).

\[ t\_{TS} = \frac{2.33– 1.48 }{0.295 \sqrt{\frac{1}{5} + \frac{1}{5}}} = 4.56\]

There are 8 degrees of freedom, so (Table A.2) the critical value $t= 2.31$

(P=0.05)

Since the experimental value of $ltl$ is greater than this the difference between the two results is significant at the 5% level and the null hypothesis is rejected.

In fact since the critical value of 1 for P < 0.01 is about 3.36, the difference is significant at the 1% level.

ln other words, if the null hypothesis is true the probability of such a large difference arising by chance is less than 1 in 100.

\subsection{Pvalue - the observed significance}

P-value - the observed significance

We base our inference on

$|t\_{TS}|$ > t\_{1-\alpha/2}$

where $t\_{1-\alpha/2}$ is the quantile of t-distribution (often referred to as the Critical Value).

The distribution of

$t$ is given by the graph

\begin{verbatim}t=seq(-4,4,by=0.01)

plot(t,dt(t,8))

\end{verbatim}

So we reject $H\_o$ hypothesis if the observed $t$ falls to the

tails of distribution.

One can ask how much is left in the tails of distribution if

the observed value is used instead of the quantile.

This value is called the observed significance or $p$-value

We reject the null hypothesis if the p-value is smaller than $\alpha$

Example One in \texttt{R}

\begin{verbatim}

t=seq(—4,4,by=0.0l)

plot (t,dt (t, 8))

2\*pt (4.56, 8,lower.tail = FALSE)

\end{verbatim}

xone=c(55,57,59,56,56,59)

xtwo=c(57,55,58,59,59,59)

t.test(xone,xtwo,var.equal=FALSE)

\subsection{Non-equal variances case }

ln order to test $ H0: \mu\_1=\mu\_2$ when it cannot be assumed that the two samples come from populations with equal standard deviations, the statistic t is calculated, where

\[ t = \frac{\bar{X}\_1 - \bar{X}\_2}{\sqrt{ \frac{s^2\_1}{n\_1} + \frac{s^2\_2}{n\_2} }} \]

With degrees of freedom

\[ t = \frac{(\frac{s^2\_1}{n\_1} + \frac{s^2\_2}{n\_2})^2 }{\frac{s^4\_1}{(n\_1)(n\_1^2)} + \frac{s^4\_2}{(n\_2)(n^2\_2)} } \]

with the value obtained being truncated to an integer.

Non-equal variances case — Example

The data below give the concentration of thiol (mM) in the blood lysate of the

blood of two groups of volunteers, the first group being 'normal' and the second suffering from rheumatoid arthritis:

Normal: 1.84, 1.92, 1.94, 1.92, 1.85, 1.91, 2.07

Rheumatoid: 2.81, 4.06, 3.62, 3.27, 3.27, 3.76

(Hanford, J. C., Brown, D. H., McConnell, A. A., McNeil, C. j., Smith, W, E.,

Hazelton, R. A. and Stunock, R. D. 1983. Analyst 107: 195)

The null hypothesis adopted is that the mean concentration of thiol is the

same for the two groups.

The reader can check that:

Group & sample size & Mean $ Std. Deviation \\

Norm. &n\_1= 7 &\bar{X}\_1 = 1.921 & s\_1 =0.076 \\

Rheum. &n\_2=6 & \bar{X}\_2 = 3.465 & s\_2= 0.440\\

Substitution in equation (3.4) gives $t = -8.48$ and substitution in equation (3.5) gives 5.3, which is truncated to 5. The critical value is $t\_5$ = 4.03 $(P = 0.01)$ so the

null hypothesis is rejected: there is sufficient evidence to say that the mean

concentration of thiol differs between the groups.

xone=c(1.84,1.92,1.94,l.92,l.85,l.9l,2.07)

xtwo=c(2.81,4.06,3.62,3.27,3.27,3.76)

t . test (xone, xtwo)

\subsection{Paired t-test}

Table 3.I Example of paired data

Batch UV spectrometric assay Near-Infrared reflectance spectroscopy

1 & 84.63 & 83,15\\

2 & 84.38 & 83.72\\

3 & 84.08 & 83.84\\

4 & 84.41 & 84 20 \\

5 & 83.82 & 83.92\\

6 & 83.55 & 84.16\\

7 & 83.92 & 84.02 \\

8 & 83.69 & 83.60\\

9 & 84.06 & 84.13\\

10 & 84.03& 84.24\\

(Trafford, A. D., Jee, R. D., Moffat, A. C. and Graham, P. 1999. Analyst 124: 163)

ln order to test the null hypothesis, we test whether $\bar{d}$ differs significantly from 0 using the statistic $t$.